Chapter 2

Contextual Review

2.1 Introduction

In this chapter, the relationship between the geoid and an ellipsoid is reviewed and methods for the determination of the shape of the geoid are stated and compared. Historical progress in the determination of the geoid is reviewed and suggestions are made as to the future utility of the astrogeodetic geoid.

2.2 Geoid Determination

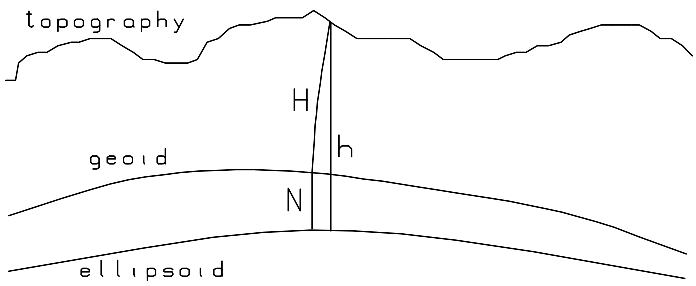

Figure 2.1 Relationship between geoid and ellipsoid.

The surveyor and engineer usually require height to be the height above the geoid or height above Mean Sea Level. Heights from GPS are found with respect to the ellipsoid. To be able to convert heights above the ellipsoid (h) to heights above the geoid (H) it is necessary to know the separation (N) between the ellipsoid and the geoid. See Figure 2.1, above. The height above the ellipsoid is measured along the normal to the ellipsoid. The height above the geoid, orthometric height, is measured along the plumb line from the point in question to the geoid. The two lines are not coincident but are very close to one another such that the approximation:

$$h \approx H + N$$

is normally without significant error.

The geoid and an ellipsoid may be considered as reference surfaces. An ellipsoid is defined mathematically. Placing it with respect to the real earth is a problem of datum definition. The geoid is a physical surface and may be defined as that equipotential surface that most closely approximates to mean sea level in the open oceans.

To find the form of the geoid using measurements of gravity it is necessary to make assumptions about the density of the earth between the topography and the geoid. Density does vary but is very difficult to measure. To avoid the problem Molodensky suggested an alternative approach. In Molodensky's model, the distance between the ellipsoid and the topography, h, is made up of two parts, the normal height from the ellipsoid to the telluroid, H^N, and the height anomaly from the telluroid to the topography, ζ. See Figure 2.2, below.

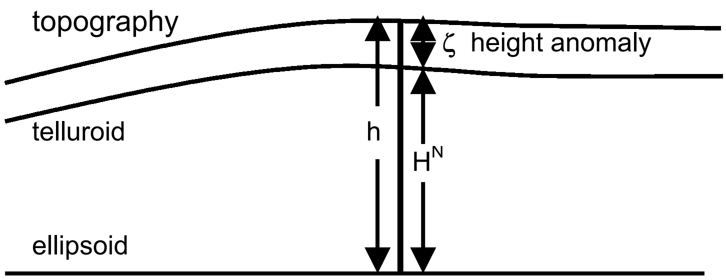

Figure 2.2 Relationship between telluroid and ellipsoid.

Therefore the relationship is:

$$h = H^N + \zeta$$

where H^N the normal height from the ellipsoid to the telluroid,

Compare this with the relationship of the geoid to the orthometric height on the previous page.

A new surface may now be plotted that is the distance of the height anomaly above the ellipsoid. This surface, the quasigeoid, is identical with the geoid in the open oceans and very close to it elsewhere, see Figure 2.3, below.

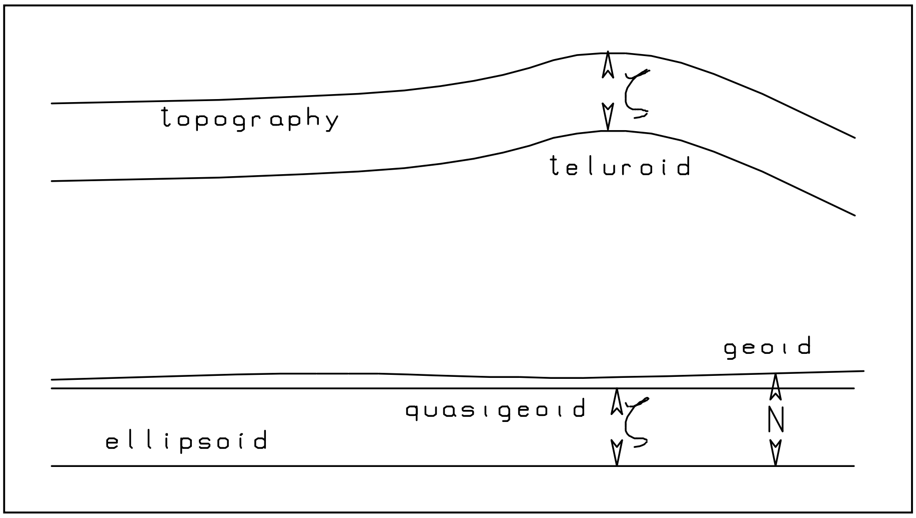

Figure 2.3 Relationship between geoid and quasigeoid.

However, it is not a level surface and so has neither geometrical nor physical meaning. It is merely a convenient surface that is relatively easy to define and to compute and is close to the geoid. However, the quasigeoid and normal heights are used extensively in Eastern Europe and are becoming more used in Western Europe as well.

2.3 Geoid Methods

There are four general methods using terrestrial observations by which a geoid may be determined. They are GPS with levelling, classical gravity methods, remove-restore gravity methods and astrogeodetic levelling. The methods involve quite different observational techniques, data processing routines, theoretical approximations, compromises and quality of output. A fifth method involves the study of satellite orbits to determine the potential field of the earth from which a smoothed, world best fitting, geoid may be derived. Such studies lead to best absolute geoids, but because of excessive smoothing, relative precision is less good.

2.4 GPS and levelling

The use of GPS with precise levelling is probably the easiest way to establish a local relative geoid. As with many observations in surveying and geodesy, it is much easier to be precise, relatively, than absolutely. If height above the ellipsoid, h, can be found from GPS and height above Mean Sea Level or the geoid, H, (or the quasigeoid) can be found from levelling and gravity observations, through geopotential numbers, then the separation is simply given by

$$N = h - H$$

If the separation is found at a suitable number of points in the area of interest, then a model of the separation may be formed. However, from GPS what we really get is relative height above the ellipsoid, and from levelling difference in orthometric height. Therefore, the following equation better reflects the situation:

$$N_1 - N_2 = h_1 - h_2 - (H_1 - H_2)$$

where the subscripts refer to the points at the end of the GPS baseline. $h_1 - h_2$ comes from GPS, $H_1 - H_2$ comes from levelling and therefore to find N_2 we must first know or assume a value for N_1, that is, at the origin of the survey the separation must be defined. Therefore, all subsequent values for separation are relative to the origin. Similarly, all values of orthometric height, derived from GPS observations and geoid model heights, will lead to orthometric heights relative to the assumed orthometric height of the origin.

If GPS is to be used to find orthometric (or normal) heights to the best available precision then a GPS and levelling derived geoid is unlikely to be sufficiently precise. A numerical example illustrates the problem. If difference in height between two points by levelling/gravity may be found to ± 0.01m and difference in height above the ellipsoid may be found by GPS to ± 0.015m then the difference in separation may be found to

$$\sigma_{\Delta N} = (0.01^2 + 0.015^2)^{½}\,m = 0.018m$$

If this value is now used to find the orthometric height of a point with a different set of GPS observations then the formula becomes

$$H_1 - H_2 = h_1 - h_2 - (N_1 - N_2)$$

and the precision of the orthometric height difference is

$$\sigma_{\Delta H} = (0.015^2 + 0.018^2)^{½}\,m = 0.023m$$

Thus the orthometric height solution using GPS and a derived geoid model, in this example, is worse by a factor of 2.3 compared with a straightforward levelling solution, and by a factor of $^{0.023}/_{0.015} = 1.5$ compared with the difference of ellipsoidal height found from GPS.

To avoid the problem of an orthometric difference height solution which is significantly worse than the quality of the GPS value for ellipsoidal difference in height, it is necessary to use a relative geoid model that is significantly better than the quality of the observed/computed GPS difference heights. This of course cannot be obtained from GPS and levelling.

2.5 Classical Gravity Methods

There are two approaches to the problem of creating a geoid from observed gravity data, those of Stokes (Stokes, 1849) and Molodensky (Molodensky et al, 1962). The derivation of the formulae for the Stokes solution is complex and may be found in Heiskanen & Moritz (1979) and Vanicek & Krakiwsky (1982) among others. Stokes' Formula leads to the determination of the geoid, while that of Molodensky leads to the quasigeoid.

2.6 Stokes' Formula

Stokes Formula is derived from a consideration of a solution to the *geodetic boundary value problem*, that is, the determination of the figure of the earth. From gravity measured on the surface of the earth the separation of the geoid and the ellipsoid can be derived, if a number of quite significant assumptions can be accepted. Simply stated, Stokes' Formula is:

$$N' = \frac{R}{4\pi\gamma_m} \iint_\sigma S(\psi)\, \Delta g \, d\sigma$$

where

$R = (a^2.b)^{1/3}$ = radius of sphere of equal volume to the ellipsoid. a and b are the semi-major and semi-minor axes of ellipsoid.

γ_m = mean gravity of the ellipsoid

Δg = free air anomaly at angular distance ψ

$S(\psi) = \mathrm{cosec}(½\psi) - 6\sin(½\psi) + 1 - 5\cos\psi - 3\cos\psi \log_e(\sin(½\psi) + \sin^2(½\psi))$ (Stokes' Function)

ψ = the angular separation of the observation and computation points at the centre of the erath

$d\sigma = \cos\phi \, d\lambda \, d\phi$, a surface element

The free air anomaly is defined as:

$$\Delta g = g + F - \gamma$$

where g, the magnitude of the gravity vector, is measured on the surface; γ is computed on the ellipsoid; and the free air reduction, F, is

$$F = -\frac{\delta g}{\delta h} h \approx -\frac{\delta \gamma}{\delta h} h \approx 0.3086 \text{ h mGal} \quad \text{(h is in metres)}$$

The free air reduction assumes that there are no masses external to the geoid. The masses to be removed are assumed to be all below the observation point. In practice, there will be nearby mountains or hills higher than the observation point and valleys below the observation point. Both reduce the magnitude of the local gravity vector. The correction for this terrain or “orological” effect is further described in Bomford (1980) and Heiskanen and Moritz (1979). World-wide gravity therefore requires a world-wide terrain model. In fact, if all external masses were removed to infinity that would change the mass of the earth and move the geoid.

As a result Stokes’ Formula does not find the geoid but more strictly the separation of the co-geoid, N', from the earth mass centred ellipsoid used in the computation of the normal gravity, γ.

The Stokes' Formula above requires integration over the surface of the earth. Gravity observations are made at discrete points and so the formula needs to be adapted for a practical solution, as a summation rather than integration process. The practical application of formula is:

$$N' = \frac{R}{4\pi\gamma_m} \sum_{-\pi/2}^{\pi/2} \sum_{0}^{2\pi} \Delta g \, S(\psi) \cos\phi \, d\lambda \, d\phi$$

In practice, the earth is divided into a number of blocks. The nearer the block is to the computation point for N', the smaller it is made because errors in the computed value of N' are more sensitive to errors in gravity anomalies nearest to the point of computation. The values of N' are with respect to the earth mass centred reference ellipsoid as used in the standard gravity formula which was used to calculate the gravity anomalies. The reference ellipsoid must have the same rate of rotation and same mass as the earth or a "zero-order" undulation will occur.

Global gravity observations are required. Global gravity data of sufficient precision does not exist and therefore there will be systematic errors in any gravimetric geoid. However, the absence of a complete block of data on one side of the world will have a near constant effect over a block on the other side. In practice, dense gravity data is required over the area (country) of interest and some distance beyond. For the UK, this will require significant amounts of marine gravity.

Systematic errors in the block mean anomaly can occur at higher altitudes. The free air reduction is 0.3086 mGal/m. This means that it is necessary to measure height each time gravity is measured. If levelling data of sufficient accuracy already exists then there should not be a problem. If height data does not exist and height transfer is made with GPS over long ranges, of say 100km, then an error of up to 4m in the relative assumed orthometric height could occur. This would lead to an error of over 1 mGal in the computed block mean anomaly.

The method is based upon a "spherical earth", therefore there are approximations relating to the earth's flattening, i.e. of the order of 0.0033. This is systematic over a small area and will not exceed 0.3m.

One point requires global gravity coverage, in theory at least. Once global gravity coverage has been obtained then it can be used to create a global geoid.

Stokes' Formula can give the detailed shape of the geoid in a local area but will contain major systematic errors. Astronomy, see Section 2.9, below, can give precise values of the slope of the geoid without significant systematic error. Therefore a combination of astronomical and gravity data can lead to a precise local relative geoid.

2.7 Molodensky's Solution

For the reduction of measured gravity on the surface, it is necessary to assume the density of the masses below the point, as in the free air reduction in Stokes Formula. Molodensky's solution for "geoid" determination avoids this problem. The geometric height is expressed as

$$h_P = H^N{}_P + \zeta_P$$

so

$$\zeta_P = h_P - H^N{}_P$$

$$= h_P - h_Q$$

where P and Q are as defined in Figure 2.4. The gravity anomaly is now defined as

$$\Delta g = g_P - \gamma_Q$$

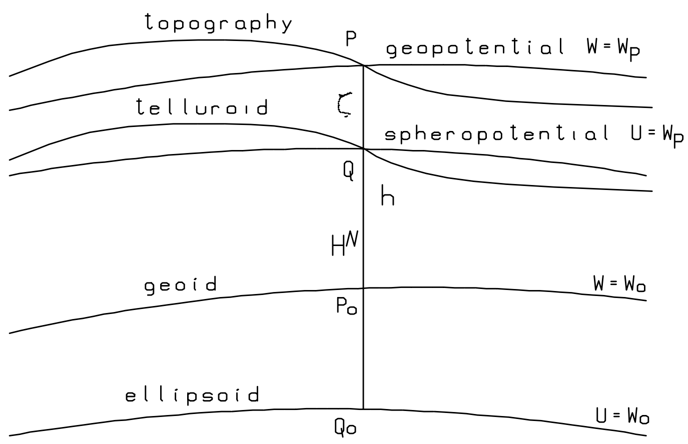

Figure 2.4 The relationship between geopotential at ground surface and spheropotential at telluroid.

This is quite a difference in definition. Note in particular that there is no assumption about density involved in this. The gravity anomaly is now the difference between observed gravity on the ground and computed normal gravity on the telluroid. This can be found from a free air reduction applied upwards to normal gravity on the ellipsoid. A direct formula for computing γ_Q is given in Heiskanen & Moritz (1979):

$$\gamma_Q = \gamma_o \left\{ 1 - 2(1 + f + m - 2f\sin^2\phi)\frac{H^N}{a} + 3\frac{H^{N2}}{a^2} \right\}$$

The solution for ζ_P is similar to that of Stokes; in fact, the Stokes' Formula forms the first part of the Molodensky solution. Bomford (1980) quotes the following as the Molodensky Formula.

$$\zeta_P = \zeta_0 + \zeta_1 = \frac{R}{4\pi\gamma_m}\iint_\sigma S(\psi)\,\Delta g\, d\sigma + \frac{R}{4\pi\gamma_m}\iint_\sigma S(\psi)\, G_1\, d\sigma$$

where Δg has been defined on the previous page and G_1 is given by

$$G_1 = \frac{R^2}{2\pi}\iint_\sigma \frac{(h_A - h_P)}{r^3}\left(\Delta g + \frac{3\gamma_m\zeta_0}{2R}\right) d\sigma$$

where

h_A is the height of the observed anomaly point or, more practically, the block centre.

h_P is the height of the point where the height anomaly is to be computed.

r is the distance between the observation and computation points.

The full Molodensky Formula has further terms but they are very small and have been neglected.

Both the geoid from Stokes and the quasigeoid from Molodensky are correlated with the topography but the numerical value of the quasigeoid is in general greater than that of the geoid. At sea, of course, the geoid is coincident with the quasigeoid. The difference between the geoid and the quasigeoid may be expressed as

$$\zeta - N = H - H^{N}$$

$$= \frac{C}{g} - \frac{C}{\gamma}$$

$$= \frac{\gamma - g}{\gamma} H$$

where g and γ take their mean values along their respective lines. An approximate formula from Heiskanen & Moritz (1979) is:

$$\zeta - N \approx 10^{-7} \bar{H} H \text{ metres}$$

where

$\bar{H}$ is the mean height of the area in metres

H is the height of the point in metres

Some examples of approximate values are in Table 2.1, below.

Table 2.1 The difference between ζ and N at four sample points.

Feature	height of point	mean height of area	ζ - N
Mt. Everest, Nepal	8848 m	5000 m	4.4 m
Mt. Blanc, Switzerland	4807 m	3000 m	1.4 m
Ben Nevis, Scotland	1344 m	800 m	0.11 m
Brown Willy, Cornwall, England	419 m	100 m	0.004 m

2.8 The Astrogeodetic Geoid

The slope of the geoid is the same as the deviation of the vertical at a given point in a given direction. The deviation of the vertical may be found from a comparison of astronomical and geodetic latitude and longitude. There are a number of ways in which astronomical position may be found; of these “Position Lines”, as described by Robbins (Robbins, 1976), is the most efficient field survey method because both astronomical latitude and longitude are found at the same time. Geodetic position may be found by conventional survey means or by GPS.

The astrogeodetic geoid may be found by observing deviations at a series of stations. Deviation is the slope of the geoid and, if separation is known at least at one point, then it may be estimated at all other points within the area of the observing stations. It can take most of a single night to complete the astronomical observations for a precise determination of deviation of the vertical by conventional means at one point.

There are methods other than Position Lines, but no single other method determines both components of position simultaneously. The minimum necessary observations for Position Lines are those of precise zenith angle, and precise time. The determination of the astrogeodetic geoid is described in detail in Chapter 9.

2.9 Remove-Restore Gravity Methods

A more recent approach to the problem involves using the “remove-restore” technique with least squares collocation. The basis of the approach is to use Stokes’ integral applied to gravity data but as it may not be practical to use apply the integration over the whole of the earth the long and short wavelength components of the geoid are computed separately using a "remove-restore" approach.

Gravity anomalies are computed from the long wavelength component from a global geopotential model such as EGM96 (EGM96, 1996). The gravity anomalies are then subtracted from the raw gravity data. The remaining gravity anomalies are then used to compute the short wavelength component of the geoid height. The long wavelength component of the geoid height computed from the global geopotential model is then restored and finally the total geoid height is computed as the sum of the short and long wavelength components.

In computing the short wavelength component of the geoid from local gravity data least squares collocation may be used. The quality of local solution is highly dependant upon the quality of the local gravity data used which is why a good digital elevation model (DEM) for the computation of the terrain correction is important.

2.10 Space Based Methods

There have been a number of campaigns to determine the geoid using satellite-derived data.

Direct measurements of the lake and sea surface may be made by satellite altimetry. If the position of the satellite is known, e.g. from onboard GPS then a comparison of the satellite height above the ellipsoid, derived indirectly from GPS, coupled with altimetry data can, in principle, lead to a measure of the height of the sea surface. With knowledge of the disturbances of the sea surface such as wind, tides currents, temperature and density sea surface height can be related to the geoid. Such methods do not work well over land where knowledge of the geoid is more often needed. Programmes have included ERS-1, launched by the European Space Agency in July 1991 and the current TOPEX/Poseidon satellite being replaced now by Jason-1 (JPL, 2002).

Alternatively, the study of satellite orbital perturbations leads to knowledge of the gravity field. For the best results, the satellite needs to be close orbiting to be subject to the greatest effect of the irregularities in the earth's gravity field. However any satellite with an altitude of less than about 300km will suffer excessive atmospheric drag and its orbit will soon decay. A number of other disturbing forces such as Electromagnetic force perturbation, solar radiation, tidal perturbations and relativistic effects will also need to be taken into account (Vanicek and Krakiwsky, 1982). An analysis of orbital perturbations leads to the determination of spherical harmonic coefficients of the earth's gravitational potential field. Satellite laser ranging is a method of direct measurement to a space vehicle. It has been used with a number of satellites with corner cube prisms such as Lageos, the laser geodynamic satellite launched by NASA in 1976, the French satellite, Starlette the Japanese satellite Ajisai and the Russian satellites Etalon I and II. (SLR, 2002).

A third approach is to measure the motion of one space vehicle relative to another. With two space vehicles in the same orbit, but with different arguments of perigee, the distance between then will be affected by local gravity variations. Again, the same difficulties with minimum desirable and minimum practical altitude will apply. The new GRACE mission (GRACE, 2002) uses this approach.

A further development will be to use a gravity gradiometer and this is the main component of the future GOCE mission (GOCE, 2002).

2.11 Comparison of methods

The methods described above have their advantages and disadvantages and therefore their specific applications.

Space based methods are able to give global data and therefor a global geoid model. In all cases, except satellite altimetry the geoid model lacks short wavelength resolution because the satellites must orbit the earth at several hundred kilometres, any less and their orbits will decay rapidly. With satellite laser ranging the limitation is that the technique works well over water but less well over land. In all cases, there are enormous amounts of data to manage especially where the measurements are to be used to compute spherical harmonic coefficients.

GPS with levelling is the simplest method but does rely upon good existing orthometric height information. If there are significant errors in the height data then the geoid model will reflect those errors. Even with good height data, the geoid model will reflect the combined errors of GPS and height data.

The application of GPS with levelling for geoid model determination will be where:

- Significant orthometric height data already exists.
- A geoid of limited accuracy is acceptable.

A gravimetric geoid model requires, theoretically, worldwide gravity and height observations. In practice, gravity measured over a limited area but in extent well beyond the area of the proposed geoid model is required. For islands, this will involve the observation of significant amounts of marine gravity. Gravity observations require specialist equipment, i.e. gravimeters. Gravity magnitude values on land are easy to observe and require no more than a short stay at each point with a gravimeter. The height information does not have to be as precise as that for GPS with levelling but interpolation of map contours is unlikely to be acceptable. Although data capture is simple, processing the data to derive a gravimetric geoid is rather more complex, especially where the data is sparse. A gravimetric geoid derived with limited gravity data, especially near or in the area of the gravimetric geoid model, will contain significant biases and systematic errors.

The application of gravity for geoid model determination will be where:

- The geoid model is required for a large area, eg 100s of km square.
- A national or world geoid model is required.

- Significant amounts of gravity data already exist and computing facilities are available.

An astrogeodetic geoid model requires time-consuming astronomical observations, which can only be carried out on relatively cloud-free nights. Geodetic position is required, ideally to better precision than that obtained by astronomy and this may easily be found with GPS. The computing process to derive deviation from astronomical observations is moderately tedious. Deviation is the same as geoid slope and therefore needs further processing to derive geoid height. Precise orthometric height information is not required. The only requirements for height are for the reduction of latitude for plumb line curvature for which contour interpolation will normally be acceptable and for topographic-isostatic reductions for which a local elevation model would normally be required.

The application of astronomy for geoid model determination will be where:

- The geoid model is required for a limited area and,
- Significant systematic errors in the model are not acceptable and,
- Astronomical observations for position already exist or may be observed and,
- A precise solution is required.

In practice, geoid models of large areas are obtained using a combination of all available data, including satellite orbit and altimetry data.

2.12 Progress in the determination of the geoid

The quality of geoid and related solutions has improved significantly over the past 20 years.

Torge & Denker (1998) reviewed that Wolf in 1948 calculated an astrogeodetic geoid for Central Europe based upon 100 deflections of the vertical and achieved an accuracy of several metres. Tani in 1949 achieved a similar precision with 100 gravity anomalies. In both cases, the quality of solution reflects the sparseness of the data.

An unpublished Polish astrogeodetic geoid of 1961 is reported by Lyszkowicz (1991) as having a contour interval of 0.5 m and an estimated relative accuracy of ±0.15 m over 100 km.

In Australia, an early, 1967, astrogeodetic geoid using 600 astrogeodetic observations probably had an accuracy of ±6 m, Kearsley & Govind (1991). Subsequently an astrogravimetric geoid with contour interval of 1 m, Fryer (1972), with 1200 astrogeodetic and some gravity observations had an uncertainty of ±3 m.

Monka et al (1978) contains three short papers on, (1) a gravimetric geoid for the North Sea, (2) a GEOS-3 altimeter geoid of the German Bay and (3) an astrogeodetic geoid around the North Sea. (1) is on land and sea, (2) is at sea and (3) on land. Agreement between (1) and (2) is ±0.2 m RMS. Agreement between (1) and (3) varies from ±0.5 m RMS to ±1.4 m RMS. Astrogeodetic data is limited, in (3) but data is very dense in parts, in particular 2 groups of sites in Germany where stations are less than 5km apart. Stations in Norway are up to 120 km apart. There is little more than a single line of stations in UK.

Torge & Denker (1998) review that Levallois & Monge in 1978 had improved the Central European Astrogeodetic geoid to an accuracy of 1 to 3 metres with about 1000 deflections of the vertical.

In Poland (Lyszkowicz 1991) the 1983 situation, based upon astrogravimetric data, is little improved at ±0.10 to ±0.15 m per 100 km. However, Torge & Denker (1998) report that The European Astrogravimetric geoid EAGG1 (Brenecke et al 1983) with about 5000 vertical deflections had an absolute accuracy of ±0.9 m and relative accuracies ranging from ±0.3 m per 100 km to ±1.1 m per 1000 km.

Tziavos & Arabelos (1991) show that surface gravity with the OSU86F geopotential model gives absolute geoid heights at ±0.5 m and 2-3 ppm for relative heights in North America. Agreement with deviation data in Greece is better than ±2”. After removal of geopotential model and terrain model effects, the standard deviation of a gravity anomaly set is ±32 mGal with point accuracy of 2 to 5 mGal. Astronomical deflections ($\Delta\xi$, $\Delta\eta$) after removal of geopotential model effects are ±5”.5.

The local quasigeoid for Hungary, based on OSU86F geopotential coefficients and gravity anomaly data, Ádám & Denker (1991), has 0.2 m contour intervals.

Rapp & Nikolaos (1991) report that the OSU89A/B potential coefficient models of degree 360 have absolute agreement of ±0.59 m with the GEOSAT implied undulation and ±1.60 m with Doppler results. Relative agreement with a precise European traverse is 0.25 m ±3.5 ppm.

A gravimetric quasigeoid of Scandinavia and Finland, (Forsberg, 1991), uses a DTM with 0.5 km to 1 km resolution for terrain corrections and the OSU89B geopotential model. Agreement with a 2000 km GPS levelling traverse is ±0.10 m RMS with local relative agreement at ±0.03 m to ±0.05 m RMS over approximately 20 km (≈ 1.5 to 2.5 ppm).

Comparison of the German 1989 quasigeoid, derived from 440000 blocks of gravity data, with GPS levelling (Denker, 1991) shows RMS differences of ±0.01 m to ±0.06 m at distances of 10 to 800 km.

Fukuda & Segawa (1991) report on a Japanese geoid based on satellite altimeter and surface land and ship gravity data, with 0.2 m contour intervals.

A geoid for Italy derived from surface gravity and the IfE88E2 geopotential model, Benciolini et al (1991), has agreement of ±0.1 m to ±0.3 m with GPS and levelling.

The UNB ’90 geoid uses 266000 land and 323000 sea gravity values with the GEM-T1 geopotential model up to degree 20. Accuracy is estimated by Vanicek et al (1991) to be ±1 ppm.

Bürki and Marti (1991) report on three geoid models for a Swiss geoid, which agree to ±0.1 m to ±0.3 m. They are gravimetric, astrogeodetic and astrogravimetric. The Ivrea zone of Switzerland shows anomalies of the order of 170 mGal and deviations of 35”. A plot of an astrogeodetic geoid has 0.2 contour intervals but geoid height accuracy is claimed to be ±0.03 m. GPS results, with the Bernese software, show agreement to ±0.1 m to ±0.15 m over all of Switzerland.

Kearsley & Govind (1991), report on an Australian gravimetric geoid constructed from 430000 land and offshore gravity points for which limited tests show a comparison with GPS/levelling of ±1.7 ppm.

Torge & Denker (1991) suggest possible improvements in the existing European geoid/quasigeoid. All the methods involve improved geopotential models, global topographic-isostatic models, regional DTMs, satellite altimetry, sea surface topography models, GPS and levelling. There is no mention of astronomy presumably because of the assumed slow rate of data capture.

By 1996, astronomy as a data source appears to have been abandoned. In the whole of Segawa et al (1997) there appears to be not one mention of it. All efforts for geoid determination are based on world geopotential models, gravity, altimetry and GPS with levelling, in spite of the respective limitations of the individual methods.

Jiang (1997) reports on the geoid for France. A gravimetric geoid was adjusted to fit 1081 GPS/levelling points. The accuracy of the geoid is estimated to be 2-3cm in plane areas and 4-7 cm in mountainous areas with a relative accuracy of 2ppm over

20km, 0.2ppm over 300 km and 0.06ppm over 1000km. This latter statistic suggests a relative accuracy of 6cm over 1000km. In Belgium, Pâquet (1997) reports that the BG96 Belgian geoid has an absolute accuracy of 3-4cm and a relative accuracy of 1-2ppm up to 50km and 0.3-0.5ppm up to 300km. Similar levels of accuracy are also claimed by Denker et al (1997) for the whole of Europe for the European Gravimetric Quasigeoid EGG96. The accuracies are ±1-5cm over 10 to a few 100km and ±5-20cm over a few 1000km when compared with independent GPS/levelling data. In the Nordic and Baltic region of Europe, Forsberg et al (1997) report a fit of their geoid to ±10cm across the region but with a fit at the 1cm level in Denmark where the topography is relatively flat and the gravity coverage is particularly good. In Latvia, Kaminskis (1997) reports that there is an ±8cm agreement between the gravimetric geoid and GPS/levelling. In all the cases in this paragraph, terrain models, altimetry, tide-gauge readings, geopotential models and GPS/levelling have been taken into account.

The need for high-resolution terrain models is emphasised by Kühtreiber (1997) if geoid accuracy is to be obtained at the ±1cm level in mountainous regions.

In Australia, Zhang & Featherstone (1997) investigate the effect of the terrain correction as it affects the AUSGEIOD93 free air co-geoid. They conclude that the application of the terrain correction only reduces the RMS discrepancy from ±0.428m to ±0.410m even though the terrain effects affect the geoid by up to 0.69m. It is clear that there must be other sources of systematic error yet to be accounted for. Featherstone et al (1997) discuss the tasks for improving the Australian geoid, including considerations with respect to terrestrial gravity and terrain data, geodetic datums, geopotential models and GPS/levelling on the Australian Height Datum.

In Canada, the gravimetric geoid GSD95, as reported by Véronneau (1997), has a precision of ±5-10cm over 10s of kilometres when compared with GPS/levelling. A geoid with a precision of ±2.5cm is planned for the year 2000.

The Japanese Geoid, JGEOID 93, as reported by Fukada et al (1997) shows an RMS difference between the gravimetric and GPS/levelling geoid of ±7cm. Takana (1997) determines a local geoid with GPS and conventional but unspecified survey in the region of a volcano in Japan but only to ± 10 cm.

Marson et al (1997), working in the Ross Sea Antarctica, show a relative geoid plot with 0.25m contours based upon marine gravity, a topographic model and the remove-restore technique applied to the OSU91 geopotential model.

By contrast, in South America, Blitzkow et al (1997), report that the gravimetric geoid of South America when compared with Doppler and GPS/levelling points shows a mismatch with geoids derived from OSU91 and WGS84 of the order of ±2m. Similarly, there is a poor absolute geoid in Indonesia, a country of 16000 islands. There is a shortage of marine gravity and the uncertainty is still at the 0.7m level.

With the exception of the centre of the Western Europe, there is as yet, in 1998, no geoid that can match the relative accuracy of GPS ellipsoidal heighting.

By 1998, the EGG97 quasi-geoid model for Europe (EGG97, 1997) had been published and much of Vermeer and Ádám (1998) is concerned with its evaluation. The EGG97 was computed from 2.7 10^6 gravity measurements and 7 10^8 heights. Even so there are areas of insufficient data at sea, Torge & Denker (1998). Several of the papers are concerned with testing the EGG97 with GPS and levelling. Some incompatibility in terms of significant amounts of bias and tilt are reported. Bias is of no consequence when the geoid model is used for relative GPS heighting, but tilt is of concern. Denker (1998) reports an RMS discrepancy in Lower Saxony of ±0.038m with bias and ±0.013m with bias and tilt over 300 km, an RMS discrepancy in France of ±0.128m with bias and ±0.080m with bias and tilt over 1000 km and an RMS discrepancy through Europe of ±0.294m with bias and ±0.175m with bias and tilt over 3000 km. Since most GPS work is undertaken over short ranges this is approaching acceptable values. However, when GPS height differences can be routinely computed at the ±0.005 m level then this quasi-geoid will need to be improved.

In Hungary, Kenyeres & Virág (1998) find, after bias and tilt is removed, a fit of ±0.063 m between GPS and levelling and the EGG97.

In Denmark, a relatively flat country, a 1-cm geoid for most of the country is reported by Forsberg (1998). In Finland, also relatively flat, Ollikainen (1998) reports a fit of the FIN95 geoid to GPS/levelling of ±0.060 m, but a worse fit with other geoid models.

In Australia AUSGEOID93 is reported by Zhang et al (1998) to achieve a national fit of ±0.33m and a local fit of ±0.04 m to ±0.18 m with GPS/levelling. The national fit is a significant improvement upon that reported by Zhang & Featherstone (1997) for the AUSGEOID93 co-geoid.

In Israel, Sharni (1998) reports on a pilot project for a geoid model of a small area of 570 km^2. When compared with GPS/levelling the result has a standard error of

±0.040 m. The project has been hampered by insufficient gravity coverage, especially off the western coast of the project area.

Overall, in Torge & Denker (1998) astronomy has a higher profile than in Segawa et al (1997). "The Recomputation of the Austrian Astrogeodetic Geoid", Heiland et al (1998), is a refinement of previous work and incorporates a denser DTM of 50 x 50 m compared with the previous 350 x 350 m. The improved DTM model leads to changes of over 0".5 in some vertical deflections. The Austrian astrogeodetic geoid is seen not as a product in its own right but as a by-product of the process to produce the "Austrian Geoid 2000" which will use the 50 x 50 m DTM, 30 000 gravity observations and 700 deflections of the vertical. The accuracy of the Austrian astrogeodetic geoid is not stated.

In describing a 1-centimetre local geoid in Croatia, Colic et al (1998) used 18 vertical deflections observed with a Zeiss Ni2 astrolabe, with GPS and precise levelling. The Airy-Heiskanen model for isostasy (p135 of Heiskanen & Moritz (1979)) has been used with fixed values for $\Delta\rho$ and the depth of the Mohorovicic discontinuity. The quality of vertical deflections is ±0".5 to ±0".75 and ±0.020 m for GPS/levelling and it is claimed that this leads to a geoid model of ±0.015 m but over a limited area of less than 1000 km^2. The limitation of the knowledge of variation in surface density is acknowledged.

For Switzerland, Marti (1998) describes the CHGEO97 geoid model that has been derived from 600 vertical deflections, 70 GPS/levelling stations and a 25 m DTM. 20000 available gravity observations were not used because they did not improve the quality of the solution, probably because of insufficient rock density data. The result has uncertainty of ±0.03 m to ±0.05 m depending upon the flatness of the terrain. The CHGEO97 quasigeoid was compared with the EGG96 quasigeoid and the fit varied from ±0.05 m to ±0.15 m for flat and extreme mountainous areas respectively. Updated figures for the CHGEO98 geoid model are given in Marti (2000).

Ming (1999) reported on recent advances in the geoid determination by airborne gravimetry but current technology is limited to 5 cm local geoids.

Blitzkow et al (1999) describe the current state of progress towards a South American geoid that currently is at the decimetre level. Clearly, development across the entire world is not consistent and there are areas where considerable work is still required.

In South East Asia a regional gravimetric co-geoid is reported by Majid et al (1999) where limited sets of gravity data are used to produce a co-geoid of 0.4 metre RMS fit to GPS/levelling. The solution is claimed to be better than the Earth Gravity Model 1996 (EGM96) and OSU91A geoids in the 40º latitude by 50º longitude window under investigation.

In the UK (Iliffe 2000), evaluation of the OSGM91 geoid in a limited area suggests that relative geoid model heights are of the order of 0.013 to 0.026 m for distances between 1 and 10 km. This is now approaching the level of precision that is desirable for use with GPS.

It was recognised by Featherstone in Western Australia that a gravimetric geoid model does not allow the accurate transformation of Global Positioning System (GPS) ellipsoidal heights to Australian Height Datum (AHD) because of the effect of local geological structures, the availability and quality of gravimetric data, and the possibility of distortions in the AHD (Featherstone 2000). A solution combining GPS and AHD heights was used to adjust the gravimetric geoid so that it provided a model of the separation between the AHD and the GRS80 reference ellipsoid. An improved model of the AHD–GRS80 separation was found at the ±8 mm level in comparison to the gravimetric geoid fit of ±128 mm.

Smith (Smith et al 2000) describe problems associated with the determination of a geoid for Florida USA. They find airborne gravity data unreliable and marine gravity introduces tilts and biases to the gravimetric geoid. The test of the geoid is against GPS/levelling data but there is no overall quality statement relating to the Florida geoid model.

In the determination of a gravimetric geoid for Hong Kong, Yang and Chen (Yang and Chen 2001) use the EGM96 geopotential model with the remove-restore technique, a limited data set of 600 gravity points and a 500 metre DTM to create a gravimetric geoid. 31 checkpoints from GPS/levelling show agreement to ±20 mm.

The US National Imagary and Mapping Ageny (NIMA) and NASA's EGM96 is now considered by many as the current definitive model for practical applications. It is available as a spherical harmonic coefficient file and a correction coefficient file to calculate point geoid undulations (EGM96, 1996). The Geoid Height File consists of a 15 minute grid of point values. The EGM96 Geopotential Model to degree and order 360 has been used. The WGS 84 defined constants of ellipsoid semi-major axis and eccentricity were used to define the reference ellipsoid. In constructing the EGM96, surface gravity data was used to give detail where satellite information was insensitive, that is where degree is greater than 40. Satellite to satellite tracking data from Topex/Poseidon, EP/EUVE, and GPS/MET was used. Altimeter data from GEOSAT, TOPEX/POSEIDON, and ERS1 was used to provide detail where degree is greater than 70. Conventional tracking data included observations by SLR to Lageos2, Stella and GFZ-1 and from TRANET (Doppler) HILAT and RADCAL tracked satellites (EGM96, 2001).

The UCL Geomatics Engineering Research page states the absolute value of the WGS84 geoid height computation using EGM96 as better than metre world-wide (UCL, 2002). Relative values will of course be better.

As the area over which the geoid is to be determined decreases, problems associated with a lack of gravity data become more apparent. Although local precision becomes greater, tilts and biases in the model, due to an absence of gravity outside the immediate area of interest, become more problematic.

By way of illustration, Figure 2.5 shows a part of the geoid model currently available for UK. It is associated with the ETRS89. Figure 2.5, below, was constructed on a 10km by 10km grid at 100 metre intervals, 10000 points, using the “co-ordinate converter” at Ordnance Survey (2001) which in turn uses OSGM91. The area is centred on the village of Ravenshead in Nottinghamshire. The “contour interval” is 0.05 metres. The smooth contours probably reflect a lack of resolution in the original data set used to construct the geoid model.

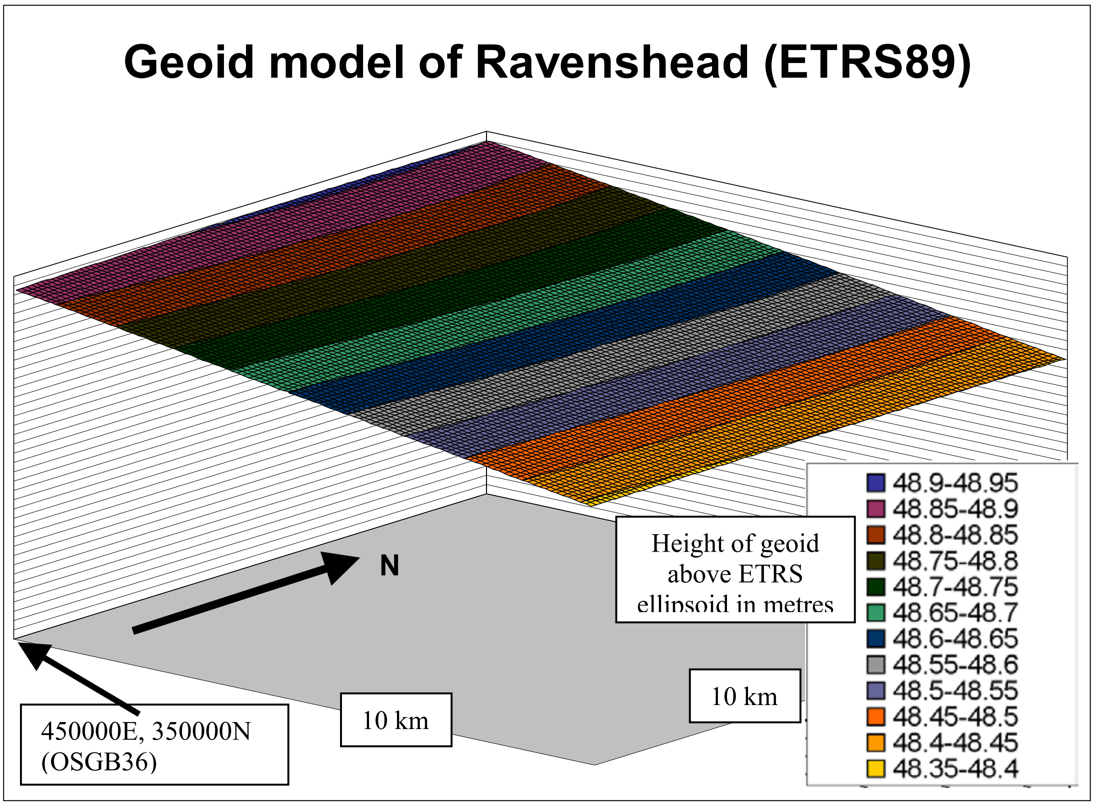

Figure 2.5 A geoid model for Ravenshead

2.13 Data Compression

Barthelmes & Dietrich (1991) describe the use of point masses for the approximation of a gravity field. This is a form of data compression where 3568 Bouguer anomalies are represented by 200 "buried masses". The quality of the gravity model reduces from an RMS of ±2 mGal to ±2.6 mGal. The compression is of 3568, 3D points (2D position plus Bouguer value), to 200, 4D points (3D position plus mass size), and so represents a data compression ratio of 1:13.4 with quality loss ratio of 1:1.7 (≈ 1:2.62/2.2). The overall gain is therefore 1:7.9.

2.14 Astrogeodetic data capture

Rowe et al (1985) describe a star sensor developed by the US Defense Mapping Agency and Ball Aerospace Systems in 1984 that uses a temperature controlled, 256 x 256 pixel, charge injection device (CID). A substantial amount of observational data is required because each pixel has a 7”.7 field of view. Observing time is 2 to 3 hours and position solutions for φ and λ of ±0”.15 and ±0”.15secφ are claimed.

Bürki and Marti (Bürki and Marti 1991) report on Swiss geoid models, including an astrogeodetic one. The astronomy was observed with a “Hannover-type” zenith camera. Verticality was controlled with 2 Talyvel III levels; there is no statement of the quality of the astronomic positions.

Eissfeller & Hein (1994) report another study of the potential use of a charge-coupled device (CCD) camera system. The theoretical study considers the use of a 2048 pixel square chip mounted in a zenithal Cassegrain reflecting telescope with a focal length of 1.5 m and a 0.14 m aperture. As with all CCD approaches, there is a problem of relating the azimuth and altitude of the principal axis of the telescope to the azimuth and altitude corresponding to an individual pixel. Their system appears to need a cooling system to ensure electronic stability of the sensor chip. In spite of the size and complexity of the technology, for portable field use their expected accuracy is still no more than ± 0”.3 in position, a result achievable by manual means.

Buerki (1998) in a private communication describes work over a 15-year period leading towards an automated position line capture system using a transportable zenith camera system. He believes that the resulting accuracy of the geoid over the Alps is within 1 to 3 cm for most of Switzerland. He asserts that the results clearly indicate that deflections of the vertical are very well suited to determination of the geoid in mountainous regions. In the Geodesy and Geodynamics Laboratory at the Institute of Geodesy and Photogrammetry in the Swiss Federal Institute of Technology, they operate a zenith camera and electronic theodolites with a software package that enables on-line observation and computation of position. The hardware includes a time digitising unit, 1 pps input from a time signal or GPS-receiver, providing epochs defined by a manual stop switch with a resolution of a few milliseconds and a “steering Notebook-PC”. The software uses precise computations of apparent places of stars in the FK5. The estimation of position does not appear to use least squares but the method is claimed to produce results within one hour, although the precision is not stated.

Wei (1999) describes the adjustment of an astrogeodetic and GPS network of China that includes 48519 astrogeodetic points. This indicates that there is an extraordinary amount of astronomical data in existence in China. Whether that is just from Laplace stations, or whether there is a similar amount of positional data, is unknown but, if there is, then it could be used for the computation of a Chinese astrogeodetic geoid.

Hui et al (1999) describe how an earthquake affected the local plumbline in Yunnan, China in 1995.

2.15 Data quality comparison

Comparing the utility of gravity and astronomic data is not a simple task. The quality of a geoid derived from a single data type will depend upon:

- Quality of the observations.
- Available observation density or cost of new data capture.
- Reduction of the observations.
- Theoretical assumptions in the use of the reduced observations to create the geoid.

2.14.1 Quality of the observations

Gravity and astronomy use measurements of the magnitude and direction of the gravity vector respectively. The full range of possible gravity measurements at sea level is approximately 5000 mGals. A semi-circle above the horizon is 64800”. If the two can be compared, then 1 ppm of the full observational range is both 5 μGals and 0”.65. Current instrumentation is of the order of both precisions, though single observations in the field are unlikely to be this good.

2.14.2 Available observation density or cost of new data capture

There is currently much more gravity data available than astronomical data. New gravity data is observed in traverses and several points per operator may be observed in a day. The previous limitation on astronomical data capture has been the observing time required at a point, which has been of the order of several hours. With an electronic theodolite and the method described in Chapter 6 of this thesis the point data capture time could approach that of gravity.

As a questioner at the IUGG99 conference at Birmingham in July 1999 noted, deflections of the vertical are sensitive to local gravity variations but these can be filtered by terrain modelling to reduce the sensitive astronomic determinations to smoothed gravity based determinations. Gerstbach (1999) supports the view that fewer plumb line deflections than gravity observations are required to compute a geoid of equal precision. He quotes the need for astronomical points every 5-10 km or gravity observations every 1-3 km to obtain a 1-cm geoid in Austria. He does not state the basis for these figures. If correct, they imply that over a given area between 11 and 25 times more gravity than astronomical observations are required. The time taken to collect data consists of travel time to and from site and observation time on site. This indicates that even with longer on-site data collection time, astronomy could be significantly more economical.

2.14.3 Reduction of the observations

Correction of gravity observations requires position for earth tide corrections, correction for drift, and a one-dimensional network adjustment for best internal fit. Reduction of gravity observations to gravity anomalies requires precise knowledge or observations of height. The approximate reduction of 0.3086 mGal/m means that a height uncertainty of ±0.01 m leads to a further ±3 μGal uncertainty. For large or national networks, this can be very demanding. The factor 0.3086 mGal/m depends upon the assumed value of density of the underlying masses being known to the same precision as the height. Height data will be required for the calculation of the terrain model used for the orological correction.

Correction of astronomical observations requires knowledge of earth rotation parameters for polar motion and approximate height for the downward continuation of the vertical for the correction for latitude. A DTM of the surrounding area will be required for a topographic-isostatic reduction where there is significant altitude variation in the surrounding area. Systematic errors in the star almanac can lead to systematic errors in astronomical position. A zenith instrument observes a limited range of stars for which the declinations are approximately the same as the latitude of the point under investigation. An instrument/system that observes stars away from the zenith will be less vulnerable to star almanac systematic errors but must properly account for the effects of zenith distance related refraction. Both systems will be vulnerable to the small amounts of systematic refraction that may exist in the zenith at the time of the observations, especially if the observations are taken over a short period.

2.14.4 Theoretical assumptions in the use of the reduced observations to create the geoid

Stokes' and Molodensky's Formulae are integrals and are replaced in practical work by summations over meaned blocks of gravity anomalies. As free air anomalies are correlated with height, then a representative anomaly at the mean block height must be found. Block sizes must be small near the computation point or there will smoothing of the computed geoid. If any theoretically worldwide data is missing, there will be errors in the computed geoid especially in points near the missing data. These effects are reduced, but not eliminated, by using geopotential models and "remove and restore" techniques such as were used in the construction of many of the geoid models reported in Rapp and Sansò (1991) and Vermeer and Ádám (1998).

Astronomical deviations give the slope of the geoid, not the geoid height itself. The geoid must either be modelled as a series expansion or by conventional astrogeodetic levelling carried out to determine the geoid height at points other than at the origin of the network.

Astronomy gives a much more direct measure of the geoid than gravity. For a relative geoid over a moderately flat and limited area of interest, for example the UK, an astrogeodetic geoid will be much easier to determine than a gravimetric one.

2.15 Plumb Line Variations

It is generally believed that there are significant and detectable non-tidal variations in the plumb line. Barlik et al (1999) report that non-tidal variations of up to 0".1 have been detected by astrometric and gravimetric means at sites in Poland and China. In one Chinese case, the variation has been associated with an earthquake where variations of up 0".1 were detected before the event. This suggests that the detection of non-tidal plumb line variations might be used as a tool for earthquake prediction. In such a case, the non-tidal variation in the plumb line could be because there has been some form of change in the earth, as suggested by Barlik et al (1999). In that case, the astrometric and gravimetric results should agree. If however the surface has moved without significant changes below the surface, then astrometry would show a difference before and after the event but gravimetry would not. In the Chinese case mentioned above, there was significant, but not complete, correlation.

2.17 The Problem with Astronomy

Astrogeodetic levelling has been used in the past for the geodetic control of national mapping. It is little used today because the astronomy part is labour intensive. The collection of papers edited by Rapp and Sanso (1991) which, although entitled "Determination of the Geoid Present and Future", is concerned only with gravity and GPS, and makes no mention at all of astrogeodetic techniques. The author of this thesis notes that the problems associated with practical astrogeodetic levelling are many. For example, with a Wild T2000 theodolite and tripod, short wave radio, stop watch, thermometer and barometer and an appropriate almanac such as the Star Almanac for Land Surveyors (SALS), (HMSO 1995), or Apparent Places of Fundamental Stars (APFS), Astronomisches Rechen-Institute (1995) and a calculator, a solution with a standard error of 3" of latitude and the equivalent in longitude (100 m) can be found. The method is very slow: it may involve two full nights' observations with a party of two per station. The author notes from his experience of astronomy that it has the following drawbacks:

- There is a danger of accidentally stopping the stopwatch by pressing the wrong button in the dark.
- Nights are cold and antisocial when standing still in the same place all night.
- The stopwatch must be calibrated against a time signal such as RWM Moscow.
- Each observer will have a different personal equation, i.e. un-calibrated individual reaction time between observing the star and pressing the stopwatch.
- The precision of operating the stopwatch is limited to about 0.2 - 0.4 seconds.
- The time taken to find suitable stars is long.
- Stars must be balanced in azimuth and altitude.
- There is no guarantee that the star seen will be in the SALS or APFS.
- Computations for one night's observations will take one day (***if*** they work out!)
- Final solution is a graphical one, so expert interpretation is required to identify the correct solution. There is no statistical determination of the quality of the solution and therefore it is difficult to estimate the precision of the computed position.

In summary, there may be two nights of observations and one day of computations per station, leading to a position solution of ±3" (Robbins, 1976). Therefore, with so much effort for so little result, there is now little use for the process. Time was not a significant consideration before computers were readily available because other surveying and geodetic operations also took significant amounts of time.

The solution to the problem is to automate the data capture and the data processing.

Recent preliminary work in this area has been completed in the form of several BEng (Hons) Engineering Surveying final year projects at Nottingham Trent University, based upon ideas by, and under the supervision of, the author. The students' understanding and the time available have limited the projects. The approach by Brookes (1994) and Greenfield (1994) was to make observations without preparation, discover which stars had been observed and use a simplified least squares solution, that is by observation equations, to find position. This represented a first attempt at Nottingham Trent to bring positional astronomy into the computer age. Davidson (1995) and Hayward (1995) took the project a little further in that they collected observations at several sites and, with a revised version of Brooks and Greenfield's suite of spreadsheets, made some (now shown to be erroneous) determinations of the deviation of the vertical. However, they managed to improve productivity to 3 hours of observations and 2 hours of computation per station. The solution would still only have been good, theoretically, to ±3" if the errors in the suite of spreadsheets had been corrected.

In the summer of 1995, the author conducted feasibility trials with a Wild T2000 electronic theodolite and his own Hi8 video camera for recording the image of a moving star against the theodolite graticule. The RWM Moscow radio time signal was recorded on the sound track with voice recording of the theodolite readings. RWM Moscow was used because it codes the difference between earth rotation time and Universal Time (DUT1) to 0.02s.

Internal camera time code was recorded on the picture. The time was correct to 1 video frame, that is 0.04 of a second. There was no "personal equation", that is systematic human bias in timing, because the observer was not recording the time. A spreadsheet with limited data was used to predict star positions and therefore there was no requirement to search for stars. The achieved data capture rate was five minutes per star with a single observer. Most of the five minutes was taken up with manually lining-up the camera with the theodolite telescope since both instruments were on different tripods. It is probable that this data rate could be improved to four minutes per star with an assistant, and therefore 80 minutes per station for 20 stars should be achievable.

2.18 The "Position Lines" Method

"Position Lines" is an astronomical method for simultaneously determining latitude and longitude. The method is documented in published literature such as Robbins (1976) and in Breach (1997) from which the early sections of Chapter 4 have been adapted. Other astronomical methods for determining latitude require knowledge of longitude and, likewise, other methods for determining astronomical longitude require knowledge of latitude. There is also the requirement, that stars must be well balanced to mitigate against imperfect knowledge of latitude or longitude, as appropriate. The observations for position lines are those of precise altitude, time and an approximate azimuth to the star if the computation is to include a graphical plot. Approximate values of latitude and longitude are also required, as provisional values. Observations are usually made to balanced pairs of stars at opposite azimuths and at altitudes normally greater than 35°.

Correct star co-ordinates are required for the time of observation. The positions of stars are published in various astronomical catalogues each at a specific epoch, usually "J2000". J2000 is defined as the 12 hours (midday) on 1 January 2000 in Barycentric Dynamical Time (TDB) (Seidelmann, 1992). The star's positions are required at the time of observation.

For an observing programme to be efficient, stars need to be selected so that they are balanced to minimise the effect of errors in the computed value of the refraction coefficient, collimation, latitude and longitude. See Robbins (1976).

Some small corrections need to be made to compute astronomical position. Two need further development for this thesis. One relates to the gravitational attraction of the moon as the earth and moon orbit around their own barycentre. The other relates to the determination of the topographic-isostatic effect. Although theory and formulae have been developed for this effect, the formulae are cumbersome in the extreme and a more "user-friendly" approach is required.

By 1996, astronomy as a data source was in significant decline (Segawa et al, 1997) and most efforts for geoid determination were based on world geopotential models, gravity, altimetry and GPS with levelling. One exception was in Austria, (Heiland et al, 1998) where the topography limited the availability of gravity data.

For geoid models of centimetre or less accuracy, both high-resolution terrain models (Kühtreiber, 1997) and a detailed knowledge of subsurface density (Colic et al, 1998 and Featherstone, 2000) are required.

Although centimetre level geoid models are now becoming available we appear to be some way from millimetre level geoid models. If relative geoid models are to be used in conjunction with relative heights from GPS to find relative orthometric height, then it is desirable that the errors in the geoid models do not add significantly to the overall error budget. If relative heighting from GPS or its successors improves beyond current levels then a near millimetre relative geoid will become an urgent goal of geodesy. The rest of this thesis suggests one possible way forward.

2.19 Summary

In this chapter on geoid determination, the available methods of Space based systems, GPS and levelling, gravity and astronomy were compared. Historical progress in the determination of the geoid was reviewed and it was noted that astronomy had largely gone out of fashion because of the difficulty of original data capture. However if local high precision models are required then astronomy, using a method such as that described in Chapter 6, may be the best answer. Astrogeodetic data capture techniques were reviewed and an attempt to compare the differing quality of different data types was made. In the next chapter, the methodology of such an approach is detailed.