Chapter 3

Methodology

3.1 Introduction

In the previous chapter the “Position Lines” method for the determination of astronomical position and progress in the determination of the geoid were briefly described. The overall methodology of this thesis is to develop appropriate theory relating to various aspects of the determination of astronomical position, to develop practical procedures to enable the determination of astronomical position and to investigate methods for using astronomical position to create an astrogeodetic geoid model.

3.2 Theory of various aspects of the determination of astronomical position

The theory relating to the determination of position by the technique of “Position Lines” needs to be developed. The classical graphical approach is to be reviewed and least squares based solutions considered. The least squares solutions may include: refraction effects and their rates of change; theodolite vertical collimation and its rate of change also latitude and longitude as unknowns to be solved for. Equations are derived.

Investigations of the direction of balance based on error in computed refraction coefficient and on error in computed collimation are required.

The nature, effect and evaluation of non-random errors in time are to be considered. An equation for the effect on the observed vertical angle of an error in horizontal pointing must be found. Observing and computing strategies are to be considered including; consideration of the observing parameters of star elevation and azimuth limits; start time of observations; the number of stars to be observed; the overall balance of stars in azimuth and altitude and the number of observations per star against number of stars. A simple formula to model refraction will be required.

The earth and moon orbit around their own barycentre. The barycentre is within the earth. Because of this orbit, there is a centrifugal effect that varies with time and place. It is anticipated that it will have a variable effect upon the local direction of the vertical.

The effect of the moon's gravitational attraction, and the effect of centrifugal acceleration of the observation point around the barycentric rotation axis, upon the deviation of the vertical need to be evaluated. To achieve this the relationship between the Geocentric, Orbital and Topocentric Co-ordinate systems must be established.

The variations of Lunar Right Ascension and Declination must be reviewed so that lunar co-ordinates may be used to determine the magnitude and direction of the lunar deviation of the vertical.

Variations in the Earth's mass distribution will have an effect upon the direction of the local vertical. If observations could be made at the geoid then deviation of the vertical measured at the geoid would truly represent the slope of the geoid. However observations are made on the surface and must be reduced to the geoid by the application of a correction for curvature of the plumb line in the meridian (Robbins, 1976) and for the topographic-isostatic effect (Nagy, 1966). In the latter case it is necessary to compute the difference between the sideways attraction at the point of observation on the ground and at the point of computation at the geoid. For a fully rigorous correction to be applied then variations in the density of the topography must also be taken into account. Since the topographic-isostatic effect can be of the order of several arc seconds in mountainous terrain then variations in overall density with direction may also be significant. Since the practical approach to computing the topographic-isostatic effect involves dividing the terrain into blocks and computing the sideways force then it will be necessary to find the mean density for each block by other means. Such data may not always be available.

Earth tides, due to the attraction of the sun and moon will have a small effect upon measured astronomical position. Vanicek and Krakiwsky (Vanicek, Krakiwsky 1982) state that the effect is less than 0".05, and is therefore negligible.

Ocean loading might also have a small effect in some places. It is understood that the West of Cornwall rises and falls by about 5cm with the tide. If the West Country may be considered as hinged about a meridian through Bristol then the 2½cm variation from the mid tide value over the distance from Bristol to Land's End, 220km gives a maximum discrepancy of 0".024 and therefore probably negligible.

3.3 Practical procedures to enable the determination of astronomical position

The productivity associated with a number of possible observing strategies is to be investigated. Methods of automating the observing process need to be examined and a practical process developed. Applicable methods worthy of investigation include using a photodiode for precise timing of star observations, detection of star passage over the theodolite crosshair and the determination of time of star passage over the theodolite crosshair.

Alternatively a method using a video camera may be appropriate and this will require consideration of issues relating to precise linkage of time to a video frame, the determination of the motion of the image of the star, the determination of the theodolite horizontal crosshair in the video image and the determination of the time of star image passing the horizontal crosshair image to be considered.

A practical method for the determination of precise observing time is essential. Time may be obtained from radio time signals or from GPS. With GPS and video recording of the star passage across the crosshairs, the problem to be solved is that of linking the video frames to the GPS time signal.

Updated values of star co-ordinates must be found before an observing programme or computations to find position can be made. The process necessary is to create a suitable catalogue of star data at a reference date, J2000, and to compute updated values of the co-ordinates at the epoch of interest. J2000 is the reference date for all modern catalogues.

Observing and computing processes need to be examined in some detail to find a method that will give a good solution under the specific conditions of observation. Stars must be selected in an observing programme so that they are balanced to minimise the effect of errors in the computed value of the refraction coefficient, collimation, latitude and longitude. The “productivity” of various technology combinations needs to be considered to find a process that delivers economical solutions. Automation of the observing process using a photodiode or a video camera, with or without a video capture board must be considered. Relating time to the observation will be a major issue. Theory, formulae and computational method must be considered for each case.

Astronomical position is observed on the ground surface but the slope of the geoid is computed at the geoid. The reduction of observed astronomical position includes a component to account for the different attractions of the surrounding topography on the ground surface compared with that at the geoid. The usual method of determining this topographic-isostatic effect on the ***magnitude*** of gravity observations needs to be adapted and developed for the effect on the ***direction*** of gravity observations. If a simpler method that does not require a local terrain model could be developed it would reduce the computational effort required to compute the topographic-isostatic effect.

To demonstrate the application of theory and processes developed, a practical determination of astronomical position, and hence slope of the geoid, will be required.

3.4 From astronomical position to astrogeodetic geoid model

The relationship between the geoid and an ellipsoid was reviewed in Chapter 2. Methods for the determination of the shape of the geoid by GPS with precise levelling, by gravity observations and by astrogeodetic methods were compared. Historical progress in the determination of the geoid is reviewed and suggestions as to the future utility of the astrogeodetic geoid are put forward. Previous methods of astrogeodetic data capture were reviewed and an attempt was made to compare the usefulness of gravity and astronomical data.

In Chapter 9 the classical method for the determination of the Astrogeodetic Geoid is reviewed. The density of observations needed for an astrogeodetic geoid model is considered in terms of Kaula’s rule of thumb. An alternative consideration is developed by finding the effect of a hemispherical mountain on the deviation of the vertical and using this to create three test models of simulated topography of “Mountains”, “Hills”, “Lowlands” and a “Plain”. Geoid models are then developed and examined using “Polynomial Coefficients” deduced from observations by least squares. A similar investigation is made using “Interpolation of Deviations”. Finally a scheme for the development of a geoid model by “Progressive Nodes” is proposed.

The first stage, now, is to develop “Position Lines” theory to make it applicable for practical applications using least squares methods and this is the subject of the next chapter.