Chapter 5

Star Positions

5.1 Introduction

For the theory developed in Chapter 4 to be applied, it is necessary to find updated values of star co-ordinates so that an observing programme and later computations to find position, can be made. The process is to create a suitable catalogue of star data at a reference date, J2000, and to compute updated values of the co-ordinates to the epoch of interest.

5.2 J2000 Catalogue Construction

Several different catalogues were investigated for their usefulness. The Fifth Fundamental Catalogue (Fricke, 1988) was downloaded via the Internet from the US Naval Observatory at USNO (1997a). The advantage of using the Internet version of the FK5 over the printed FK5 was that there was no human error during data entry and the Internet file contains more stars with values for parallax and radial velocity. Parallax is the apparent movement of a star because of the motion of the earth around the sun and radial velocity is the rate of change of distance of the star from the earth and sun barycentre. However, the introduction to the printed FK5 contains useful formulae for the determination of precession and incorporating the effects of parallax and radial velocity.

Precession is the angular motion of the earth's polar axis around the ecliptic pole plus the absolute motion of the ecliptic pole. General precession is the combined motion and its effect on the position of the vernal equinox. Nutation is the predictable short-term deviation of the earth's axis from its long-term precession.

The Internet file contains the full FK5 whereas the printed version is in 2 publications, Pt I and Pt II. Pt II contains mostly the less bright stars.

The Internet FK5 file was edited in an Excel spreadsheet to remove all data except J2000 Right Ascension, Declination, their rates of change, star number, parallax and radial velocity. All stars less bright than those with a magnitude of 4, at J2000, were also removed. The resulting file contained 429 stars.

The FK5 is not a “bright star” catalogue. The contents of the resulting spreadsheet were compared with the Star Almanac for Land Surveyors (SALS) 1996 (HMSO, 1995) to identify which stars brighter than magnitude 4 were missing. The US Navy Observatory has several other much larger catalogues, but none of these could be downloaded because of the excessively slow load times, on readily available computers in the Department of Civil and Structural Engineering at the time of investigation (1996).

The missing stars identified in SALS 1996 were found in the printed (Yale) Bright Star Catalogue (YBSC) (Hoffleit, 1982) and for these the J2000 Right Ascensions, Declinations, their rates of change, parallax and radial velocity were manually abstracted. The rates of change of Right Ascension and Declination in the FK5 are in time seconds and arc seconds per century respectively. In the YBSC they are both in arc seconds per year. The YBSC values of rates of change were made compatible with the FK5 equivalents by transforming as follows:

$$\alpha'_{FK5} = 0.15\ \alpha'_{YBSC} \sec\delta$$

$$\delta'_{FK5} = 0.01\ \delta'_{YBSC}$$

where α is Right Ascension

δ is Declination

Since large catalogues from the US Naval Observatory could not be downloaded and edited, Right Ascensions and Declinations with more significant figures were found, one star at a time from VizieR (1997). The data set is the Smithsonian Astrophysical Observatory (SAO) catalogue. Values from the YBSC and use of the “minimum search area” setting ensured that on most occasions only the target star was found. The final result was that all stars were listed in the spreadsheet to at least $0^s.01$ in Right Ascension and 0".1 in Declination.

The distribution of stars by magnitude and catalogue is shown in Table 5.1 below.

Table 5.1 Star Magnitudes in the FK5 catalogue with additional stars from the YBSC/SAO catalogue.

Magnitude	Catalogue		
	FK5	YBSC/SAO	Total
≥ 0	4	0	4
≥ 0.5	10	0	10

≥ 1.0	16	0	16
≥ 1.5	23	0	23
≥ 2.0	48	2	50
≥ 2.5	86	5	91
≥ 3.0	157	14	171
≥ 3.5	258	27	285
≥ 3.99	429	51	480

A second larger catalogue was created to give more flexibility in the selection of stars for different investigations. This catalogue only used data from sets available on the Internet. The two selected data sets were the Bright Star Catalogue (BSC), (NASA, 1997a) and the Positions and Proper Motions Catalogue (PPM), (NASA, 1997b) which between them contain α, δ, α', δ', parallax and radial velocity at J2000 to respective places of decimal of $0^s.001$, 0".01, $0^s.0001$/yr, 0".001/yr, 0".001 and 1km/s. However, neither catalogue contains all the data to the required level. Before selecting data from the two catalogues, it was first necessary to select only those stars that were unambiguously common to both catalogues. The selection routine was as follows:

1. Remove all stars with a magnitude less bright than 7.1 from both catalogues. This reduced both catalogues to a size that could be handled in an Excel spreadsheet.
2. Remove all stars within 5 arc minutes of each other. This was to avoid ambiguous solutions in future work.
3. Remove all stars from the BSC that did not appear in the PPM, based upon agreement in α and δ, at the $0^s.1\sec\delta$ and 1" levels respectively.
4. Remove all stars from the PPM that did not appear in the BSC, based upon agreement in α and δ, at the $0^s.1\sec\delta$ and 1" levels respectively.
5. Remove all stars from the BSC that did not appear in the PPM, based upon agreement in SAO number.
6. Remove all stars from the PPM that did not appear in the BSC, based upon agreement in SAO number.
7. Remove all stars from both catalogues where the SAO and HD (Henry Draper) numbers were not the same in both catalogues.

8. Select the star number as the SAO number. Select α, δ, α‘ and δ‘ from the PPM. Select parallax and radial velocity from the BSC.

This composite catalogue was termed MCBJ2000. The distribution of stars by magnitude is in Table 5.2, below.

Table 5.2 Star Magnitudes in the MCBJ2000 catalogue.

Magnitude	No of stars
≥0	5
≥0.5	10
≥1.0	16
≥1.5	22
≥2.0	42
≥2.5	85
≥3.0	170
≥3.5	286
≥4.0	478
≥4.5	844
≥5.0	1419
≥5.5	2386
≥6.0	3886
≥6.5	6053
≥6.6	6694

In 1998 a single catalogue containing all the necessary parameters, i.e. α, δ, α‘, δ‘, parallax and radial velocity was published at NASA (1997c and 1997d). The former is in 24 files, one for each hour of Right Ascension and the latter as a single 32Mb zip file that expands to a text file of 152Mb. The SKY2000 Master Star Catalog is reported on by Myers et al (1997).

SKY2000 contains 299,485 stars, some with as faint a magnitude as 11.5. SKY2000 contains α, δ, α‘, δ‘, parallax and radial velocity at J2000 to respective places of decimal of $0^s.0001$, 0”.001, $0^s.00001$/yr, 0”.0001/yr, 0”.00001 and 0.1km/s. This represents an order of improvement in all statistics, and 2 orders for parallax, compared with the composite catalogue, MCBJ2000, described above.

Compared with the PPM catalogue, where position uncertainties were globally quoted as 0".22, there are individual uncertainties for α and δ at J2000. The first 31000 stars in SKY2000 were investigated to see if there was a relationship between magnitude and position uncertainty. The results expressed graphically are in Figure 5.1, below.

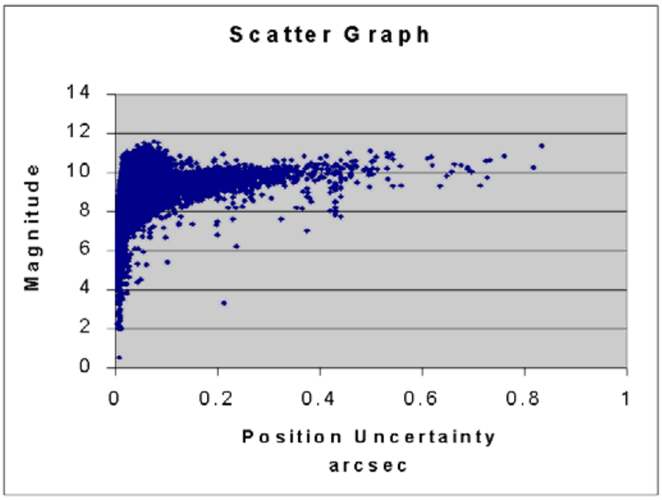

Figure 5.1 Star Magnitude and Position uncertainty for SKY2000 Master Star Catalog.

The graph gives an impression of a greater spread of uncertainties than actually exists. A better indication of the relationship between magnitude and position uncertainty can be shown from Figures 5.2, below, where the star set is divided into magnitude bands, 2.0 to 2.4999, 2.5 to 3.999 etc. A graph of the number of stars in each band against the mean magnitude of the band is on the left. The relationship between RMS position uncertainty and mean magnitude of the band is on the right. The large "spike" in Figure 5.2b for the 3.0 to 3.499 magnitude band is due to a single star with a very large position uncertainty. If that star is removed from the set then the graph becomes very much smoother. The offending star has been removed from the final sub-set of SKY2000 that is to be used for practical observations. The graphs do indicate that the RMS of position uncertainty of all stars brighter than magnitude 7.0 is better than 0".02.

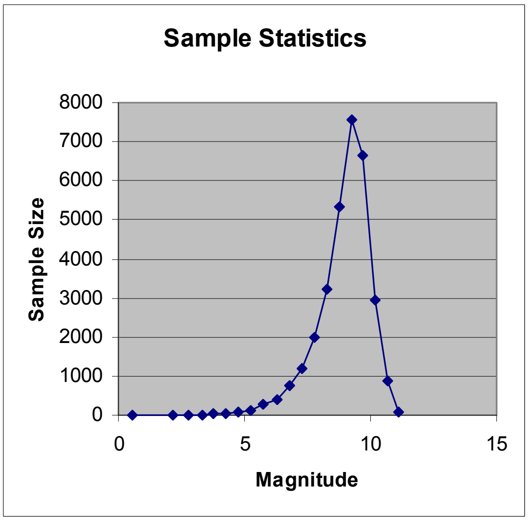

Figure 5.2a Star Magnitude and sample size for the SKY2000 catalogue.

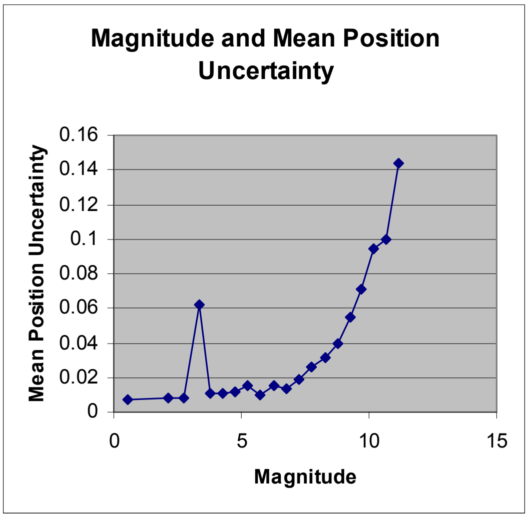

Figure 5.2b Star Magnitude and Mean Position Uncertainty for the SKY2000 catalogue.

A useful set of suitable stars from the SKY2000 catalogue was required for practical observations. The criteria for acceptance of a star were that:

There is no “distance to the nearest neighboring master catalog star no more than two magnitudes fainter than this star” listed, i.e. no close stars of similar brightness, OR

There is no “magnitude difference of brightest and second brightest components” listed, i.e. no bright double stars, OR

The magnitude of the duller component is numerically greater than 7, i.e. a double star can be accepted if the duller star is very faint, OR

The magnitude difference between the brightest and second brightest component is greater than 3, i.e. close stars can be accepted provided that there is a big difference in magnitude between them, OR

The distance to the nearest star more than 2 magnitudes fainter is less than 0º.00011 away, i.e. a pair of stars of sufficiently unequal magnitudes are so close that the visual centre of gravity of the pair will be negligibly far from the brighter star of the pair, OR

The distance to the nearest star greater than 2 magnitudes fainter is greater than 6', i.e. a pair of stars of sufficiently unequal magnitudes are so far apart there will be no ambiguity as to which star is being observed.

Since the SKY2000 catalogue appears to be at least an order of magnitude better than the composite MCBJ2000 catalogue compiled earlier, then it might be expected that the RMS difference between star positions in SKY2000 and MCBJ2000 would be of the order of the uncertainties of the MCBJ2000 catalogue, i.e. 0".2.

An arbitrary set of 1700 stars that appear in both SKY2000 and MCBJ2000 were selected and the differences in α and δ and their RMS at J2000 were found. The results are in Table 5.3, below.

Table 5.3 Co-ordinate differences between the SKY2000 and MCBJ2000 catalogues.

	Mean Difference Arc seconds	**RMS difference Arc seconds**
Right Ascension * cos(Declination), α*cos(δ)	-0.00404	0.026836
Declination, δ	0.002063	0.027242

The results indicate that the worst mean difference, i.e. in Right Ascension would lead to an East-West tilt of any computed geoid of less than 2mm in 100km (1.8mm subtends 0".004 at 100km). The RMS differences are well below the precision of any theodolite currently on the market. Therefore, it can be concluded that it will make no practical difference whether the SKY2000 or the MCBJ2000 catalogue is used. The above also shows that either the MCBJ2000 catalogue was more precise than the 0".22 suggested by the PPM catalogue or that SKY2000 is not as precise as its authors' claim.

5.3 Ephemeris Update

J2000 Right Ascension and Declination values for stars need to be updated to the epoch of interest. Since a set observations with a T2000 theodolite takes about 1½ hours then the Right Ascensions and Declinations in the middle of the set of observations will still be precise enough for the beginning and the end of the observations. The systematic errors associated with such an assumption will be largely a function of the change in short-term nutation. An estimate of the maximum systematic effect upon Right Ascension and Declination can be found as follows.

5.3.1 Error in Right Ascension

If $\delta = 0$

$$\Delta\alpha = \cos 23°.5 \ \Delta\psi$$

where $\Delta\psi$ is the nutation component in longitude

From Table I of APFS 1996 (Astronomisches Rechen-Institute, 1995) the maximum one day change in $\Delta\psi$ appears to be 0".127 (18-19 January) and therefore the maximum error in Right Ascension in ½ of 1½ hours is:

$$\Delta\alpha = \cos 23°.5 \ \ 0''.127 \ \ 0.75/24$$
$$= 0''.004$$

An error in Right Ascension translates directly as an error in longitude and when δ = 0 the error in longitude will be 0.12 m.

5.3.2 Error in Declination

If $\alpha = 6^h$

$$\Delta\delta = \Delta\varepsilon$$

where $\Delta\varepsilon$ is the nutation component in obliquity.

In Table I of APFS 1996 the maximum one day change in $\Delta\varepsilon$ appears to be 0".045 (21-22 January) and therefore the maximum error in Declination in ½ of 1½ hours is:

$$\Delta\delta = 0''.045 \ \ 0.75/24$$
$$= 0''.001$$

Both systematic errors are negligible.

In converting J2000 values to the epoch of interest, account needs to be taken of precession, long and short periods and planetary elements of nutation, proper motion, radial velocity, annual parallax, annual aberration, diurnal aberration, polar motion and gravitational light deflection.

5.3.3 Diurnal Aberration

Diurnal aberration depends, among other things, upon latitude and hour angle and is therefore best applied as a final correction to Right Ascension and Declination in the "position line" computation.

5.3.4 Gravitational light deflection

Gravitational light deflection by the sun is small but significant only if the star is near the sun. In practical terms, a worst case might be that if the star and sun are on the same observer's azimuth with the star at a zenith angle of 50° and the sun is 10° below the horizon. In this case the gravitational light deflection is less than 0".01. This can be verified by applying formula 3.261-1 of Seidelmann (1992).

5.3.5 Polar motion

Although polar motion is unpredictable and can only be included in computations processed after the observations have been taken, estimates of its value for a recent epoch can be found on the Internet. Alternatively, it can be ignored, and if all observations are taken within a period of a few days its effects upon position and hence geoid slope will be constant.

With the last three corrections omitted, at least at this stage, the remaining corrections listed above are applied as follows:

5.3.6 Precession and Space Motion

A description of precession is given in Section 3.21 of Seidelmann (1992). The precession angles ζ, z and Θ are quoted from Lieske (1979) in both Seidelmann (1992) and Fricke (1988). The formulae can be simplified as follows:

$$\zeta = 2306''.2181t + 0''.30188t^2 + 0''.017998t^3$$
$$z = 2306''.2181t + 1''.09468t^2 + 0''.018203t^3$$
$$\Theta = 2004''.3109t - 0''.42665t^2 - 0''.041833t^3$$

where t is the period in Julian centuries since J2000. In the above simplification, it is assumed that the "initial equinox" and J2000 are the same. The determination of the updated Right Ascension and Declination can be computed by means of a unit vector of direction cosines, **u**, and a precession matrix, **P**.

$$\mathbf{P} = \mathbf{R}_z(-z)\, \mathbf{R}_Y(\Theta)\, \mathbf{R}_z(-\zeta)$$

and from formula 3.21-8 of Seidelmann (1992)

$$\mathbf{P} = \begin{bmatrix} \cos z \cos\Theta \cos\zeta - \sin z \sin\zeta & -\cos z \cos\Theta \sin\zeta - \sin z \cos\zeta & -\cos z \sin\Theta \\ \sin z \cos\Theta \cos\zeta + \cos z \sin\zeta & -\sin z \cos\Theta \sin\zeta + \cos z \cos\zeta & -\sin z \sin\Theta \\ \sin\Theta \cos\zeta & -\sin\Theta \sin\zeta & \cos\Theta \end{bmatrix}$$

In Fricke (1988), **u** and its derivatives are:

$$\mathbf{u} = \begin{bmatrix} \cos\delta\cos\alpha \\ \cos\delta\sin\alpha \\ \sin\delta \end{bmatrix}$$

$$\frac{d\mathbf{u}}{d\alpha} = \begin{bmatrix} -\cos\delta\sin\alpha \\ \cos\delta\cos\alpha \\ 0 \end{bmatrix}$$

$$\frac{d\mathbf{u}}{d\delta} = \begin{bmatrix} -\sin\delta\cos\alpha \\ -\sin\delta\sin\alpha \\ \cos\delta \end{bmatrix}$$

At the epoch of interest

$$\mathbf{r} = \mathbf{P}\left(\mathbf{u}_0 + \frac{d\mathbf{u}_0}{dt}\Delta t\right)$$

where
$$\frac{d\mathbf{u}_0}{dt} = 4.8481368\ 10^{-6}\left(\mu_0\frac{d\mathbf{u}_0}{d\alpha} + \mu_0'\frac{d\mathbf{u}_0}{d\delta} + 21.094953\ \pi_0 V_0\,\mathbf{u}_0\right)$$

where μ_0 is the rate of change of Right Ascension in units of "/century

μ'_0 is the rate of change of Declination in units of "/century

π_0 is the parallax is in arc seconds

V_0 is the radial velocity is in km/s.

Right Ascension and Declination are then formed from:

$$\alpha = \tan^{-1}\left(\frac{r_2}{r_1}\right)$$

$$\delta = \tan^{-1}\left(\frac{r_3}{(r_1^2 + r_2^2)^{1/2}}\right)$$

5.3.7 Nutation

A description of nutation is given in Section 3.22 of Seidelmann (1992) where the International Astronomical Union (IAU) nutation series and set of fundamental arguments (Van Flandern 1981) are used with updates to some terms from Herring

(1987). Planetary effects upon nutation have been included even though they are very small, in the order of 0".0001 to 0".00001. The contents of the main nutation table of 530 argument multiples and 106 coefficients, each for longitude and obliquity have all been included (Table 3.222.1 from Seidelmann (1992)). The coefficients and arguments of multiples were confirmed by comparing with those on Internet page at USNO (1997b). The planetary terms are listed on pages 117-119, equations 3.224-3 and Table 3.224-2 of Seidelmann (1992).

The fundamental arguments listed at Table 3.222.2 in Seidelmann (1992), have also been used in this research. Initial computations produced Right Ascensions and Declinations that were significantly at variance with APFS 1996 (Wielen, 1994). Some "trial and error" and deductive reasoning led to the conclusion that there was an error in the formula for Ω, "the longitude of the mean ascending node of lunar orbit on the ecliptic measured from the mean equinox of date". The formula quoted in Seidelmann (1992), can be condensed to:

$$\Omega \quad = \quad 135^\circ\, 2'\, 40''.280 - 1934^\circ\, 8'\, 10''.539t + 7''.455t^2 + 0''.0008t^3$$

where t is the time since J2000 in Julian centuries.

If the first term is revised to **125**° 2' 40".280 then there is agreement between the output of this author's spreadsheet and Table II of APFS 1996 (Wielen, 1994) in the determination of $\Delta\psi$ and hence in the computation of both the long and short period terms of the equation of the equinox. The value of 125° was verified by comparing with the value of Ω computed from 1900 to J2000 using the formulae on page 544 of Anon (1976) and is also implicit in the Fortran code at IERS (2001) which is a published sub-routine of the International Earth Rotation Service (IERS), Central Bureau (IERS/CB).

The mean obliquity from equation 3.222-1 on page 114 of Seidelmann (1992) is given by:

$$\varepsilon_0 \quad = \quad 23^\circ\, 26'\, 21''.448 - 46''.8150t - 0''.00059t^2 + 0''.001813t^3$$

and the obliquity of date is

$$\varepsilon \quad = \quad \varepsilon_0 + \Delta\varepsilon$$

and so the nutation matrix is

$$\mathbf{N} \quad = \quad \mathbf{R}_x(-\varepsilon)\, \mathbf{R}_z(-\Delta\psi)\, \mathbf{R}_x(\varepsilon_0)$$

which is:

N =

$$\begin{bmatrix} \cos\Delta\psi & -\sin\Delta\psi\cos\varepsilon_0 & -\sin\Delta\psi\sin\varepsilon_0 \\ \sin\Delta\psi\cos\varepsilon & \cos\Delta\psi\cos\varepsilon\cos\varepsilon_0 + \sin\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\cos\varepsilon\sin\varepsilon_0 - \sin\varepsilon\cos\varepsilon_0 \\ \sin\Delta\psi\sin\varepsilon & \cos\Delta\psi\sin\varepsilon\cos\varepsilon_0 - \cos\varepsilon\sin\varepsilon_0 & \cos\Delta\psi\sin\varepsilon\sin\varepsilon_0 + \cos\varepsilon\cos\varepsilon_0 \end{bmatrix}$$

The terms $\Delta\psi$ and $\Delta\varepsilon$, the nutation components in longitude and obliquity, are derived from:

$$\Delta\psi = \sum_{i=1}^{n} S_i \sin A_i \qquad \Delta\varepsilon = \sum_{i=1}^{n} C_i \cos A_i$$

where S_i and C_i are coefficients and A_i are the sums of the arguments and multiples.

The nutation matrix is applied as:

$$\mathbf{r} = \mathbf{N}\mathbf{r}_0$$

where $\mathbf{r}_0$ is the **r** vector corrected for precession above and therefore precession and nutation corrections are applied in the new **r**.

5.3.8 Parallax

Diurnal parallax is negligibly small. Annual parallax for closer stars is significant. Seidelmann (1992) quotes as the corrections from barycentric to geocentric place as:

$$\Delta\alpha = \pi_0 (X\sin\alpha - Y\cos\alpha)/(15\cos\delta)$$

$$\Delta\delta = \pi_0 (X\cos\alpha\sin\delta + Y\sin\alpha\sin\delta - Z\cos\delta)$$

where π_0 is the parallax in arc seconds and X, Y and Z are the earth's barycentric co-ordinates in astronomic units.

If the earth's orbit around the sun is assumed to be circular and the effect of the obliquity of the ecliptic is assumed to be negligible, then:

$$X = -\cos\odot$$

$$Y = -\sin\odot$$

$$Z = 0$$

where $\odot$ is the longitude of the sun.

Robbins (1976) gives more useable and precise formulae for parallax. They may be reduced to:

$\Delta\alpha = \pi_0 (\sin\odot \cos\alpha \cos\varepsilon - \cos\odot \sin\alpha) \sec\delta$

$\Delta\delta = \pi_0 (\sin\odot (\sin\varepsilon \cos\delta - \cos\varepsilon \sin\alpha \sin\delta) - \cos\odot \cos\alpha \sin\delta)$

5.3.9 Application of Gravitational Light Deflection

Although the effect of gravitational light deflection is small, a correction has been applied for completeness. The formulae used are those quoted or adapted from pages 484-5 of Seidelmann (1992) and Breach (1997) for approximate Right Ascension and Declination of the sun.

$L = 280^\circ.460 + 36000^\circ.77\ t$

$G = 357^\circ.528 + 35999^\circ.050\ t$

$\lambda = L + 1^\circ.915 \sin G + 0^\circ.020 \sin 2G$

$\varepsilon = 23^\circ.4393 - 0^\circ.01300\ t$

$E = -1^\circ.915 \sin G - 0^\circ.020 \sin 2G + 2^\circ.466 \sin 2\lambda - 0^\circ.053 \sin 4\lambda$

$\alpha_\odot = GAST - UT_1 - E + 12^h$

$\delta_\odot = \sin^{-1}(\sin\varepsilon \sin\lambda)$

where L is the mean longitude corrected for aberration

G is the mean anomaly

λ is the ecliptic longitude

ε is the obliquity of the ecliptic

E is the equation of time

$\alpha_\odot$ and $\delta_\odot$ refer to the sun

Although the formulae are approximate, they are more than good enough for the purpose of determining $\alpha_\odot$ and $\delta_\odot$ for the correction for the gravitational light deflection.

Anon (1983), on page S20, gives formulae for the determination of corrections to α and δ for gravitational light deflection from which the following are derived.

$\cos D = \sin\delta_\odot \sin\delta - \cos\delta_\odot \cos\delta \cos(\alpha - \alpha_\odot)$

$$\Delta\alpha = \frac{0''.00407 \cos\delta_\odot \cos(\alpha - \alpha_\odot)}{(1 - \cos D) \cos\delta}$$

$$\Delta\delta = \frac{0''.00407 (\sin\delta \cos\delta_\odot \cos(\alpha - \alpha_\odot) - \sin\delta_\odot \cos\delta\ )}{(1 - \cos D)}$$

where:

D   is the geocentric angular separation of the star from the sun.

The above formulae lead to small corrections for α and δ. Values for α and δ for a selection of FK5 stars on various dates in 1996 were computed using a spreadsheet. The results were compared with the values listed in Wielen (1994). Short term and planetary nutation terms were not applied. The results are given in Table 5.4 below. Only seconds of time or arc are shown.

Table 5.4 Comparison of star co-ordinates between the FK5 catalogue and this author's spreadsheet.

At date and time – Upper Transit on	FK5 Star No	α from APFS time s	δ from APFS arc s	α from spread-sheet time s	δ from spread -sheet arc secs	δ dd.mm	15 Δα cosδ arc secs	Δδ arc secs
8-Jan-96	1	11.282	16.66	11.2819	16.662	29.04	0.001	-0.002
28-Mar-96	20	6.770	19.49	6.7701	19.469	30.50	-0.001	0.021
17-May-96	42	31.122	52.42	31.1225	52.412	35.35	-0.006	0.008
25-Aug-96	99	28.716	38.77	28.7155	38.747	55.52	0.004	0.023
3-Dec-96	1134	41.814	15.26	41.8139	15.259	6.57	0.001	0.001
23-Dec-96	215	34.044	-43.64	34.0437	-43.634	-34.04	0.004	-0.006
19-Jan-96	263	52.761	-50.88	52.7587	-50.883	-50.36	0.022	0.003
8-Mar-96	294	14.275	16.79	14.2742	16.797	24.24	0.011	-0.007
27-Apr-96	336	58.866	-17.19	58.8664	-17.205	-60.37	-0.003	0.015
6-Jun-96	365	57.266	25.85	57.2651	25.850	9.54	0.013	0.000
5-Aug-96	423	2.744	58.41	2.7426	58.409	15.26	0.020	0.001
24-Sep-96	481	30.345	-12.85	30.3433	-12.849	-59.39	0.013	-0.001
3-Nov-96	545	52.372	-28.50	52.3713	-28.470	-5.38	0.010	-0.030
23-Dec-96	585	26.690	-7.61	26.6890	-7.602	-3.24	0.015	-0.008
9-Jan-96	633	28.010	55.14	28.0106	55.137	9.22	-0.009	0.003
28-Feb-96	674	36.675	46.27	36.6747	46.272	29.14	0.004	-0.002
8-Apr-96	720	32.832	-39.20	32.8335	-39.194	-7.56	-0.022	-0.006
17-Jun-96	780	5.698	24.49	5.6986	24.493	33.57	-0.007	-0.003
5-Sep-96	859	24.068	4.44	24.0685	4.442	23.32	-0.007	-0.002
23-Nov-96	890	26.047	46.43	26.0478	46.435	46.26	-0.008	-0.005
						RMS	0.011	0.011

The RMS values indicate that star positions are in agreement with Wielen (1994) to 0".013. Better agreement could probably have been achieved with more precise formulae for $\odot$, the true geometric longitude of the sun. However, since the precision of the spreadsheet will lead to systematic errors of astronomic position in the order of only 0.3 m, then there is no need to pursue the matter further.

The output of this author's spreadsheet with short term and planetary nutation terms applied was tested against the output of MICA (1998). The FK5 catalogue in MICA was computed for 5 separate dates, 200 days apart and starting at 0^h 1 January 1999. The apparent co-ordinates of 547 arbitrarily chosen stars that were common to this author's spreadsheet and the FK5 in MICA for each of the 5 epochs were compared. The RMS of $\Delta\alpha \cos\delta$ and $\Delta\delta$ were respectively, 0".012 and 0".009 with an overall RMS of 0".010. These values are indication of the correctness of the formulae applied in this author's spreadsheet. A similar test was applied using selected stars from the SKY2000 Master Star Catalog (NASA, 1997d). The RMS of $\Delta\alpha \cos\delta$ and $\Delta\delta$ were respectively, 0".102 and 0".126 with an overall RMS of 0".115.

These values indicate the changes that have taken place in the J2000 values of the star co-ordinates and other parameters, from the various catalogues used in the construction of the MCBJ2000 catalogue, to those of the SKY2000 Master Star Catalog. It appears to indicate that, although MICA was published in 1998, it does not use the most precise data available at the date of its own publication.

5.3.10 Aberration

The movements of the observer and of the star through space cause an apparent displacement of the star as seen by the observer. This aberration has three components, secular, annual and diurnal aberration. Secular aberration is caused by the proper motion of the earth and of the star. It is very small, different for each star and therefore ignored. Annual aberration is caused by the earth's orbit around the sun.

<u>Annual Aberration</u>

Seidelmann (1992) quotes the following formulae for corrections to be applied to the geometric Right Ascension and Declination to get the apparent Right Ascension and Declination.

$$\Delta\alpha = -(\kappa \sin\odot + \kappa\, e \sin\Pi) \sin\alpha \sec\delta$$
$$- (\kappa \cos\odot \cos\varepsilon + \kappa\, e \cos\Pi \cos\varepsilon) \cos\alpha \sec\delta$$

$$\Delta\delta = -(\kappa \sin\odot + \kappa\, e \sin\Pi) \cos\alpha \sin\delta - (\kappa \cos\odot \cos\varepsilon + \kappa\, e \cos\Pi \cos\varepsilon)(\tan\varepsilon \cos\delta - \sin\alpha \sin\delta)$$

where κ is the constant of aberration
$\odot$ is the true geometric longitude of the sun
e is the eccentricity of the solar orbit
Π is the longitude of perigee of the solar orbit
ε is the mean obliquity of the ecliptic

κ, the constant of aberration for the standard epoch, is given in Seidelmann (1992) as 20".49552. e, the eccentricity of the solar orbit, is 0.016708617. Π is given as:

$$\Pi = 282^\circ\, 4'\, 49''.951 + 6190''.67(t-t_0) + 1''.65(t-t_0)^2 + 0''.012(t-t_0)^3$$

No formulae are quoted for $\odot$ in the part of Seidelmann (1992) that deals with aberration. However, Yallop and Hohenkerk in a later chapter (p484) of Seidelmann (1992) give the following formulae that lead to the "ecliptic longitude".

$$L = 280^\circ.460 + 36000^\circ.770t$$

$$G = 357^\circ.528 + 35999^\circ.050t$$

$$\lambda = L + 1^\circ.915 \sin G + 0^\circ.020 \sin 2G$$

where t is the number of centuries since J2000
L is the mean longitude corrected for aberration
G is the mean anomaly
λ is the ecliptic longitude

It is assumed that, at least to an acceptable approximation, $\odot$, the true geometric longitude of the sun may be replaced by λ. The other terms have already been described.

Diurnal aberration

Diurnal aberration results from the observer being on a rotating earth. Seidelmann (1992) quotes the following formulae for corrections to be applied to the mean Right Ascension and Declination to get the apparent Right Ascension and Declination.

$$\Delta\alpha = 0^s.02133\, {}^{\rho}/_{a} \cos\phi' \cos t \sec\delta$$

$$\Delta\delta = 0''3200\, {}^{\rho}/_{a} \cos\phi' \sin t \sin\delta$$

where ρ is the geocentric distance

ϕ'	is the geocentric latitude
t	hour angle
a	equatorial radius

t may be found from:

$$t = \alpha - GMT - R - \lambda$$

Since $\rho\cos\phi'$ is the distance of the station from the earth's rotation axis, that is the radius of the small circle of latitude, then the equations may be rewritten as:

$$\Delta\alpha = 0^s.02133\ {}^{\nu}/_{a}\ \cos\phi\ \cos t\ \sec\delta$$

$$\Delta\delta = 0''3200\ {}^{\nu}/_{a}\ \cos\phi\ \sin t\ \sin\delta$$

where ν, the radius of curvature in the transverse, plane is given by:

$$\nu = a(1 - e^2\sin^2\phi)^{-½}$$

and e in this formula is the eccentricity of the earth ellipsoid model. With negligible loss of accuracy geodetic values may be used in place of astronomic values for this purpose.

5.4 Star Catalogues

In the Table below is a summary of the star catalogues used during this investigation.

Table 5.5 Star catalogues used during this investigation.

Catalogue	Source or Publisher	Format	Useful data	Significant figures	Remarks
Star Almanac for Land Surveyors (SALS)	HMSO	Book	α at 12 epochs δ at 12 epochs magnitude	$0^s.1$ 1" 0.1	α and δ insufficiently precise. This is a bright star catalogue so it can be used to identify all bright stars in other catalogues. 695 stars
Fifth Fundamental Catalogue Part 1 (FK5 Pt1)	Astronomisches Rechen-Institut	Book	α J2000 δ J2000 α' J2000 δ' J2000 magnitude * parallax * radial velocity	$0^s.001$ 0".01 $0^s.001$/cy 0".01/cy 0.01 0".001 0.1km/sec	Precise catalogue. This is an astronomical catalogue with an even spread of stars but does not contain all the bright stars. 1535 stars.

Fifth Fundamental Catalogue Parts 1 and 2 (FK5)	Internet USNO, (1997a)	electronic	α J2000 δ J2000 α‘ J2000 δ‘ J2000 magnitude	$0^s.001$ 0”.01 $0^s.001$/cy 0”.01/cy 0.01	Precise catalogue. This is an astronomical catalogue with an even spread of stars but does not contain all bright stars.
Apparent Places of Fundamental Stars (APFS)	Astronomisches Rechen-Institut	Book	α 36 epoch/year δ 36 epoch/year magnitude	$0^s.001$ 0”.01	Precise catalogue based upon FK5 Pt1. This is an astronomical catalogue with an even spread of stars but does not contain all bright stars. Long period nutation is included. Corrections for short period nutation calculated. Tables for Apparent and Mean Sidereal Time to $0^s.001$ and short period terms of nutation to 0”.001. Useful to verify stars reduced from J2000 to epoch of date. 1535 stars.
(Yale) Bright Star Catalogue, 5th revised edition (BSC)	Internet NASA, (1997a)	electronic	α J2000 δ J2000 α‘ J2000 δ‘ J2000 magnitude parallax radial velocity SAO number Henry Draper (HD) number	$0^s.1$ 1” $0^s.1$/cy 0”.1/cy 0.01 0”.001 1km/sec	α and δ insufficiently precise. This is a bright star catalogue so it can be used to complete parallax and radial velocity data. Contains about 9000 stars
(Yale) Bright Star Catalogue	Yale University Observatory	Book	α J2000 δ J2000 α‘ J2000 δ‘ J2000 magnitude parallax radial velocity	$0^s.1$ 1” $0^s.1$/cy 0”.1/cy 0.01 0”.001 1km/sec	α and δ insufficiently precise. This is a bright star catalogue so it can be used to complete parallax and radial velocity data. Less easy to use than SALS to identify brightest stars.
Smithsonian Astrophysical Observatory (SAO)	Internet VizieR, (1997)	electronic	α J2000 δ J2000	$0^s.01$ 0.1”	α and δ found 1 at a time through internet address.

Positions and Proper Motions (PPM)	Internet NASA, (1997b) for PPM north	electronic	α J2000 δ J2000 α‘ J2000 δ‘ J2000 magnitude SAO number Henry Draper (HD) number	$0^s.001$ 0”.01 $0^s.0001$/yr 0”.001/yr 0.1	Comes in 3 parts PPM North, South and Supplementary, containing approximately 180000, 180000 and 90000 stars respectively with magnitudes to less than 10 though PPM Supplementary contains no stars of magnitude brighter than 7.4. Data used in conjunction with BSC.

* some stars only

5.5 Summary

In this chapter the process of creating a suitable catalogue of star data at the reference date, J2000, was reviewed. The star data needs to be updated to the epoch of interest taking account of Aberration, Gravitational light deflection, Polar motion, Precession and Space Motion, Nutation and Parallax. Spreadsheets have been created to produce updated values of Right Ascension and Declination for stars. These updated values then form a database that can be used to create star observing programmes and to compute position as described in Chapters 6 and 10.